Supply Network Formation and Fragility

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Complex production and specific sourcing

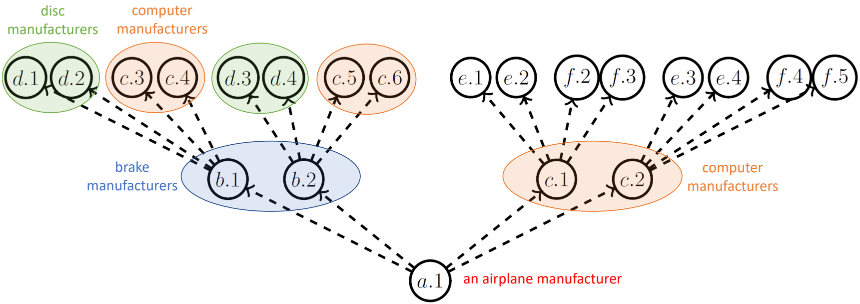
Complex production: many steps, each with **several** essential inputs.

– E.g. for an airplane these include brakes and computers.

– Brakes and computers also made of many produced inputs.

Specific sourcing: important inputs are **custom**-produced/delivered.

– Supply relationships are prone to disruption, so firms *multisource*.



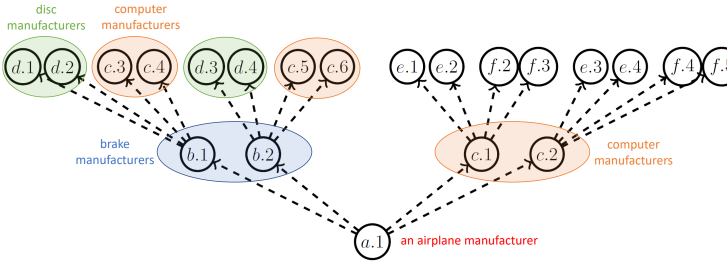
Implications for robustness

Goal: understand how production depends on relationship strength.

- Continuum of small firms involved in this kind of production.
- Network structure and relationship strength jointly determine reliability.
- Relationship strength is endogenous – firms strategically invest to make sourcing likelier, in anticipation of shocks.

Main finding: A **fragile regime** where aggregate output is arbitrarily sensitive to small, systemic shocks to relationships.

- *Precipices* where many supply chains simultaneously **freeze**.
- Not just a possibility: a **natural endogenous outcome**.

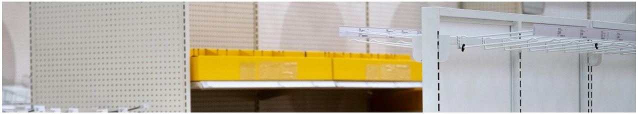


Practical motivation

How the World Ran Out of Everything

Global shortages of many goods reflect the disruption of the pandemic combined with decades of companies limiting their inventories.

The New York Times



Specialized/customized sourcing is important and getting more so.

- In normal times, the company is behind in filling perhaps 1 percent of its customers' orders. On a recent morning, it could not complete a tenth of its orders because it was waiting for supplies to arrive.

  The company could not secure enough of a specialized resin that it sells to manufacturers that make construction materials. The American supplier of the resin was itself lacking one element that it purchases from a petrochemical plant in China.

- Simply expanding warehouses may not provide the fix. Product lines are increasingly customized. The ability to predict what inventory you should keep is harder and harder.

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How can we understand this?

A global phenomenon in response to a global shock (e.g., to shipping).

- Many seemingly unrelated supply networks simultaneously affected.
- *Not* an idiosyncratic shocks/granularity situation as Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2011).
- Current network macro, e.g., Baqaee and Farhi (2019, 2020) interested in such nonlinearities (and complementarities as emphasized by Jones (2011)), but these models are really focused on **freezes**.

We take an approach based on network theory.

- Percolation models, such Elliott et al. (2014), Erol et al. (2020). Complementarity plus percolation creates very stark, new effects, cf. Buldyrev et al. (2010)

Model ingredients

A **supply network**.

Nodes are firms. A continuum $\mathcal{F}_i$ producing each product i.

Finite set of different products $\mathcal{I}$.

Technology—a producer of product i requires inputs $I(i) \subseteq \mathcal{I}$

A potential supply network $\mathcal{G}'$ of potential links (dotted lines).

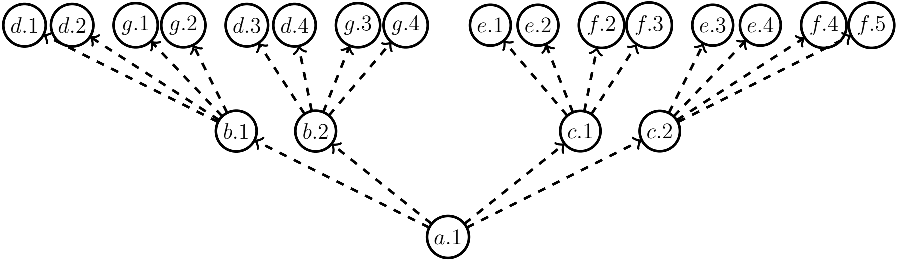
Key parameters: number of different inputs needed, number of multisourcing opportunities per firm, typical supply network depth.

A realized supply network $\mathcal{G}$ where only some of these links are operational.

Independently, with probability x.

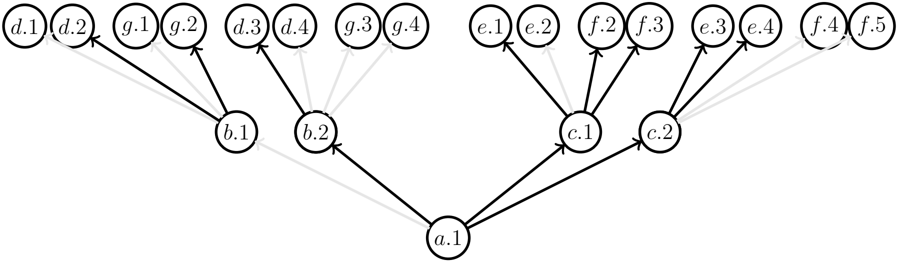
Potential supply network $\mathcal{G}'$

A graph on the set of all firms: nodes $\mathcal{F} = \{if \ : \ i \in \mathcal{I},\ f \in [0,1]\}$.
directed links $\mathcal{E}$, e.g., (if, jf')



Realized supply network $\mathcal{G}$

Each supply link in $\mathcal{G}'$ (potential supply network) is in $\mathcal{G}$ (realized supply network) with probability x, independently.
We call links in $\mathcal{G}$ operational.



Model: Supply network

Each $if \in \mathcal{F}_i$ is associated with a supply chain *depth* $d(if) \in \mathbb{Z}_+$: how many levels of customized production are needed.

We focus on the following symmetric case:

- each firm needs m distinct inputs and
- if depth $d > 0$, draws n potential depth-$(d-1)$ suppliers for each required input.
- if depth $d = 0$, can source from anyone ("generics")

Realized supply network $\mathcal{G}$

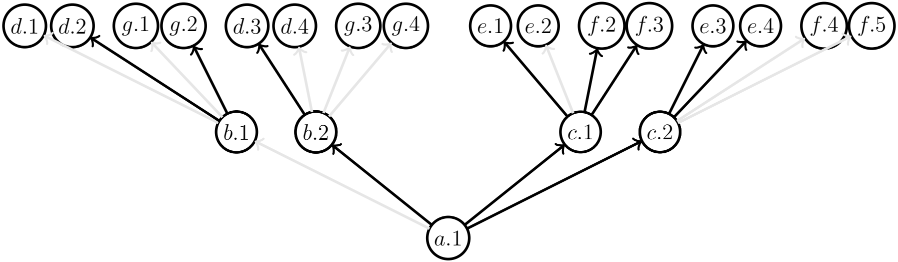


Figure: Non-operational links are in gray.
Links are operational with probability x, independently.

Depth zero firms can always function

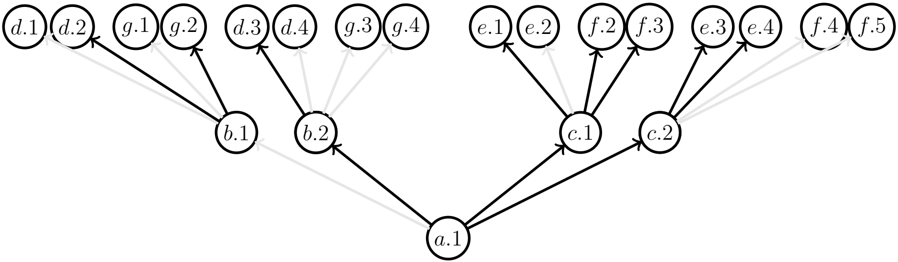


Figure: Non-operational links are in gray.

Firms requiring no customized inputs can always function.

Link failures stop some firms from functioning

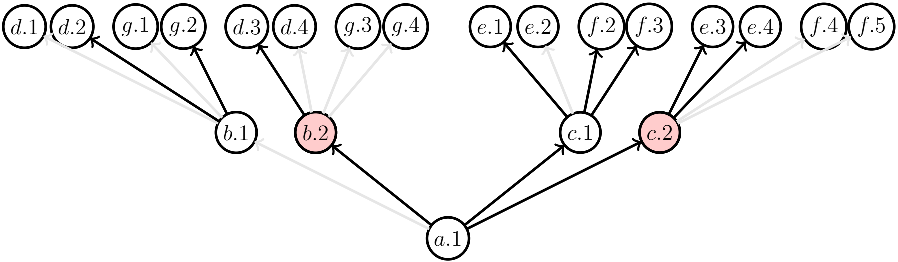


Figure: Non-operational links are in gray.
Firms at depth 1 can only function if they can source all input types.

And this stops more firms from functioning

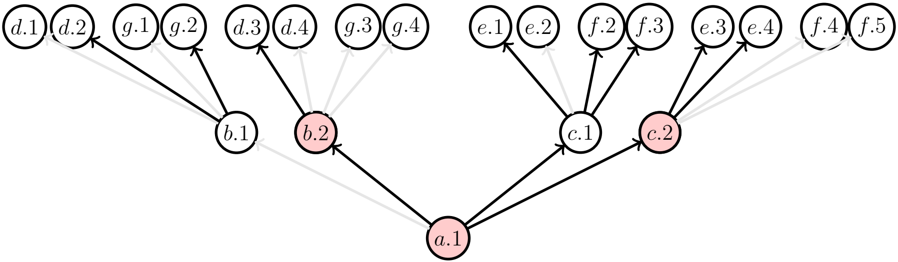


Figure: Non-operational links are in gray.
Disruptions at depth 1 can cause disruptions at depth 2.

Reliability: fraction of firms functioning

Outcome we focus on: the share of functioning firms; call it the **reliability** of the supply network, denoted by ρ.

Let $\mu \in \Delta(\mathbb{Z}_+)$ be a distribution of depths.
For this talk $\mu = \mu_\tau$, exponential distribution with mean τ.

Question 1: How does reliability $\rho(x, \tau)$ depend on x?

Why does this matter?

- Gross output is a smooth, strictly increasing and concave function of ρ.

Finding 1: When supply networks are deep, there are **precipices** where reliability (and so GDP, welfare) depend on relationship strength x arbitrarily steeply.

Interpretation: systemic shocks to relationship strength x, even very small ones, can cause arbitrarily large losses.

Reliability for a complex economy: A precipice

symmetric case: number of inputs needed for each product is $m \geq 2$
number of potential suppliers of each needed input is $n \geq 2$

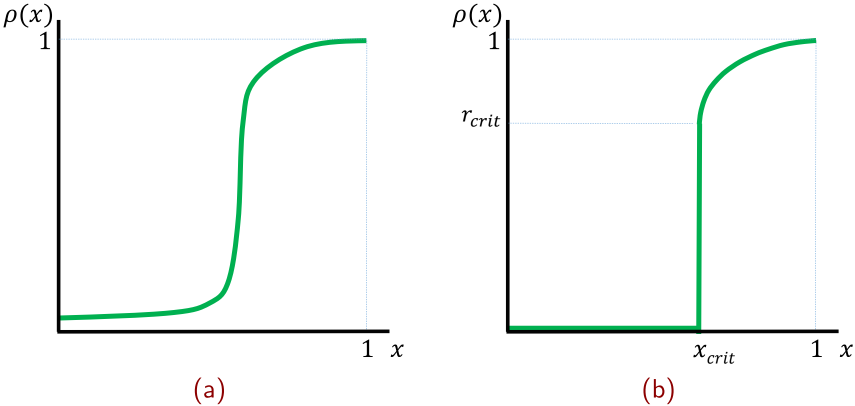


Figure: (a) $\rho(x, \tau)$: how reliability depends on relationship strength x for a particular expected depth τ. (b) the limit correspondence of the graphs $\rho(x, \tau)$ as $\tau \to \infty$.

Reliability for a simple economy: No precipice

symmetric case: number of inputs needed for each product is $m = 1$
number of potential suppliers of each needed input is $n \geq 2$

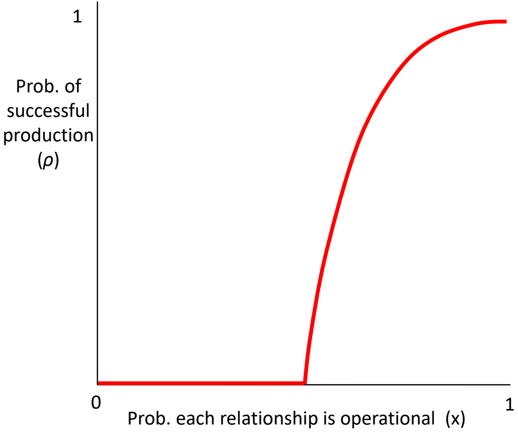


Figure: How reliability depends on relationship strength x in a simple economy ($\rho(x, \tau)$ as $\tau \to \infty$).

The precipice proposition: symmetric case

Symmetric case:

- number of inputs needed for each product is m
- number of potential suppliers of each needed input is n

Proposition

Take the symmetric case with $m, n \geq 2$ and expected depth τ. Let $\rho(x, \tau)$ be the reliability.

- For all $x < x_{\text{crit}}$, $\lim_{\tau \to \infty} \rho(x, \tau) = 0$.
- For all $x > x_{\text{crit}}$, $\lim_{\tau \to \infty} \rho(x, \tau) > r_{\text{crit}} > 0$.

- If $m = 1$, so that only one (firm-to-firm sourced) input is required, $\rho(x, \infty)$ is continuous with a kink, and bounded derivative for all x.

Takeaway

Key takeaway: Firm-to-firm sourcing in complex production yields a new source of discontinuities/strong amplification.

Questions

Recall:

> Key takeaway: Firm-to-firm sourcing in complex production environments yields a new source of discontinuities/strong amplification.

Questions:

- Does the same kind of precipice occur if the economy is heterogeneous – not symmetric? Yes.
- Does the economy end up anywhere near a precipice if the strength of relationships is endogenous?
- **Finding 2**: Yes. There is a robust force making economies of intermediate productivity fragile.

A planner's problem

$$\max_{x\in[0,1]} Y(\rho(x,\tau)) - c(x),$$

- $Y(\rho)$ is aggregate output, increasing in ρ
- x can be thought of as the quality of institutions
- $c(x)$ is a convex function—cost of maintaining institutions
- Assume $c(0) = 0$, $c'(0) = 0$, and $\lim_{x\to 1} c'(x) = \infty$.

Proposition

The planner never chooses a strength near x_{crit}—fixing any $n \geq 2$ and $m \geq 2$, the outcome x^{SP} is bounded away from x_{crit}.

Intuition: returns to relationship strength are very high near the precipice.

Model: Investment game

- Investment game: simultaneously, each firm $if \in \mathcal{F}$, makes investment $x_{if} \in [0,1]$ (probability each of its potential sourcing relationships work)
- We'll study symmetric equilibria ($x_{if} = x$ for all $if \in \mathcal{F}$)
- Timing:
  - Firms invest before the *potential supply network* is realized
  - Functional firms are determined and production occurs
- Firms' profits (from our microfoundations) can be written as:

$$\Pi_{if} = \underbrace{\kappa g(x)}_{\text{gross profit}} \quad \underbrace{P(x_{if};x)}_{\text{prob. functional}} \quad - \quad \underbrace{c(x_{if})}_{\text{investment cost}}$$

  - $P(x_{if};x)$ – probability of producing when *others* play x
  - c convex, increasing, Inada
  - $g(x)$ decreasing in x
- Which investment levels can occur in equilibrium?

Equilibrium definition

For a given κ, we say an outcome $x \in [0, 1]$ is a symmetric undominated equilibrium (SUE) if

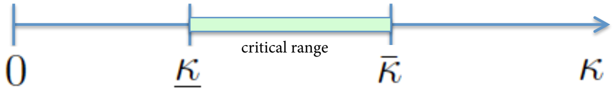
- **[each firm is optimizing]**: for gross profits $\kappa g(x)$, the investment level $x_{if} = x$ for all firms if is a Nash equilibrium of the investment game ...

$$\left(\text{i.e., } x \in \operatorname{argmax}_{x_{if}} \kappa g(x) P(x_{if}; x) - c(x_{if}) \right)$$

- ...that maximizes total surplus among the symmetric Nash equilibria. **[efficient selection]**

We denote a SUE by $x^*(\kappa)$.

Main results: Criticality



Theorem

For all τ sufficiently large there exist $0 < \underline{\kappa} < \bar{\kappa} < \infty$ such that:

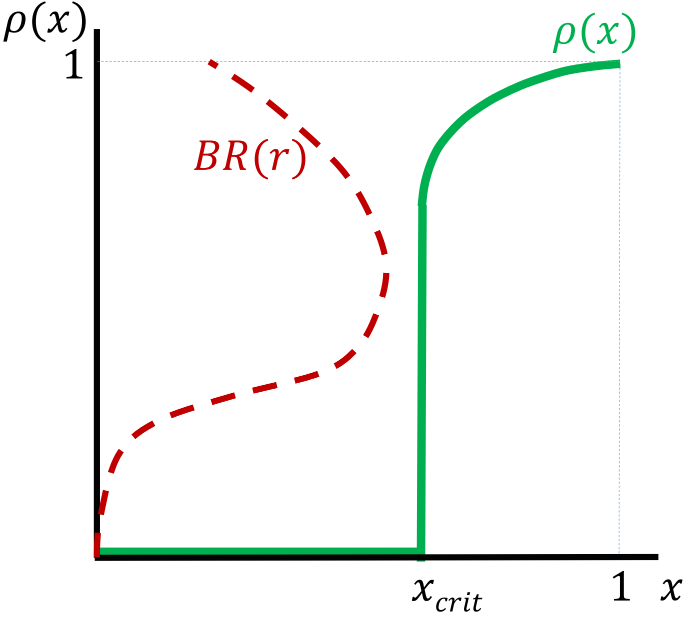
(i) For $\kappa < \underline{\kappa}$: all SUE involve 0 investment

(ii) For $\underline{\kappa} < \kappa \leq \overline{\kappa}$: $x^*(\kappa) \approx x_{\text{crit}}$ (*critical*)

(iii) For $\kappa > \overline{\kappa}$: $x^*(\kappa) > x_{crit}$ (*noncritical*)

There is a positive measure of economies that end up on the precipice.

Intuition for ranking result



Criticality implies fragility

Consider small shock to x, so that x_{if} was intended but probability of relationship operating is actually $x_{if} - \varepsilon$:

Definition (Equilibrium fragility)

- A productive equilibrium is *fragile* if *any* negative productivity shock $\varepsilon > 0$ causes the symmetric investment equilibrium to drop to $\tilde{x} < x_{\text{crit}}$ such that $\rho(\tilde{x}) = 0$.
- Else, it is *robust*.

Proposition

If $\kappa \leq \overline{\kappa}$, then any productive equilibrium is fragile. If $\kappa > \overline{\kappa}$, then any productive equilibrium is robust.

Simplest to consider unanticipated shock, but all results survive if it is anticipated and happens with small probability.

Interpretation of results

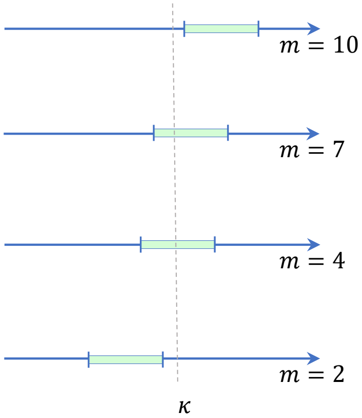
Intermediate-κ supply networks are the ones that are fragile if they are productive.

An economy has a variety of supply networks, with different m, n.

Both $\underline{\kappa}$ and $\overline{\kappa}$ **get higher as** m **increases**.

Fixing the value of κ for an economy, among products that can be produced, the more complex ones will be fragile.

Might justify intervention—e.g., pharmaceutical products might require licensing to keep G high, and investment high enough to stay away from the precipice.



Concluding comments: Applications

Aggregate fragility across economy:

- So far, focused on a single supply network, but a macroeconomy may have many separate supply networks.
- Loosely: taking a nonatomic distribution over our parameter space (e.g., κ) a positive mass of supply networks will be fragile.

Cascading failures: A transmission mechanism

- Discontinuity in one supply network may affect others through market-mediated spillovers.
- Let failure of one supply network reduce κ of other supply networks relying on its output (which they were buying on a market).
- Then failure of some fragile chains could make formerly robust chains fragile, cause them to fail as well, and so on.

Thank you!

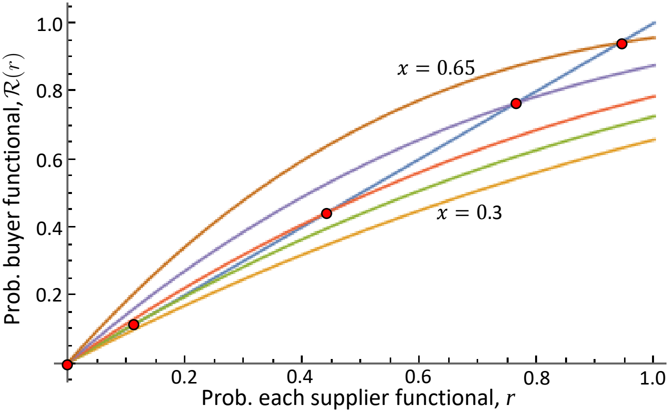
Intuition: Iteratively computing reliability

- Suppose each of your suppliers are functional with probability r independently.
- Let $\mathcal{R}_x(r)$ be the probability that you are functional.

$$\mathcal{R}_x(r) = \left[1 - \left(1 - xr\right)^{n}\right]^{m}$$

Prob. given supplier is available Prob. given supplier is not available Prob. *all* suppliers of a given input not available Prob. there is a supplier of a given input available Prob. there is a supplier of *all* inputs available Fact. For $x \neq x_{\text{crit}}$ the **largest fixed point** of $\mathcal{R}_x$ is equal to reliability as τ gets large.

Recall $m =$ complexity $\quad n =$ multisourcing number



The probability, $\mathcal{R}_x(r)$, that a focal firm is functional as a function of r, the probability that a random supplier is functional. Here we use the parameters $n = 4$ and $m = 3$. The probability, $\mathcal{R}_x(r)$, that a focal firm is functional as a function of r, the probability that a random supplier is functional. Here we use the parameters $n = 4$ and $m = 1$.

Related Literature

Network formation theory, reliability, and risk: e.g., Bala and Goyal (2000), Levine (2012), Goyal and Vigier (2014), Acemoglu, Ozdaglar and Tahbaz-Salehi (2015), Brummitt et al. (2017), Elliott, Georg and Hazell (2018), Erol (2018), Erol and Vohra (2018), Talamàs and Vohra (2018), Bimpikis, Candogan, and Ehsan (2019), Dasaratha (2020).

> Our contribution: A tractable network formation model for large complex supply networks with new features.

Complementarities in production and their implications: e.g., Kremer (1993), Blanchard and Kremer (1997), Ciccone (2002), Acemoglu, Antràs and Helpman (2007), Angeletos and Pavan (2007), Jones (2011), ...

> Our contribution: Possible concern—might actions that mitigate supply risks endogenously dampen the complementarities. We show they don't.

Related Literature

Production networks: e.g., Long and Plosser (1983), Horvath (1998), Dupor (1999), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2011), Elliott, Golub, and Jackson (2014), . . . Taschereau-Dumouchel (2017), Boehm and Oberfield (2018) and König et al. (2019), Baqaee and Farhi (2019, 2020), Acemoglu and Tahbaz-Salehi (2020).

> Our contribution: This literature focuses on smooth nonlinearities. We show how sourcing failures at the micro level give rise to discontinuities.

Self-organizing criticality and phase transitions: Jovanovic (1987), Scheinkman and Woodford (1994), an engineering/math lit. e.g., Buldyrev et al. (2010), Tang et al. (2016), and Yang et al. (2019).

> Our contribution: Fully microfounded model that shows that the most severe phase transition occurs in the most classical production network setting (once you have our kinds of failures). In our setting economy is robust to idiosyncratic shocks.

Examples of idiosyncratic disruptions

Fire at Philips Semiconductor halted production, preventing Ericsson from sourcing critical inputs, causing its production to also stop. Ericsson lost $> \$100M$ in sales, subsequently exited mobile phone business (Norrman and Jansson, 2004).

Two strikes at General Motors parts plants in 1998 led 100 other parts plants, and then 26 assembly plants, to shut down, reducing GM's earnings by $> \$2.8B$ (Snyder et al. 2016).

"It is tempting to think of supply chain disruptions as rare events. However, although a given type of disruption (earthquake, fire, strike) may occur very infrequently, the large number of possible disruption causes, coupled with the vast scale of modern supply chains, makes the likelihood that some disruption will strike a given supply chain in a given year quite high." (Supply Chain Quarterly, 2018)

Resilinc found 1,069 supply chain disruption events globally during a six-month period in 2018.

Heterogeneous supply network

No longer take the product network to be regular.

Now $|\mathcal{I}_i|$ has arbitrary cardinality m_i.

No longer take number of potential suppliers to be regular.

Now a different number $n_{ij} \geq 1$ of potential suppliers of product j for producers of product i.

No longer take link strength to be uniform

Firm if chooses how much effort to exert sourcing each input.

Proposition

Suppose all complexities $m_i \geq 2$. Let $x_{if,j} = X_{ij}(\xi)$, where $X_{ij} : [0,1] \to [0,1]$ is a strictly increasing C^1 onto function and ξ is an economywide parameter (e.g., institutional quality). There is a critical ξ_{crit} such that $\lim_{\tau \to \infty} \boldsymbol{\rho}(\xi, \infty) = 0$ for all $\xi < \xi_{\text{crit}}$ and $\lim_{\tau \to \infty} \boldsymbol{\rho}(\xi, \infty) > r_{\text{crit}} > 0$ for all $\xi > \xi_{\text{crit}}$.